PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory, Semester I 2018 - IIC3242
Homework 1
Deadline: Thursday, 22 March 2018

1 The Goldbach problem [1 point]

Consider the following string:

$$s = \begin{cases} 1, & \text{if Goldbach's conjecture is true} \\ 0, & \text{if Goldbach's conjecture is not true} \end{cases}$$

Let $L = \{s\}$ be the language containing only the string s. Is L decidable? Explain why or why not?

Note: Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states that every even integer greater than 2 can be expressed as the sum of two primes. There is still no proof for this conjecture.

2 Programming Turing machines [4 points]

The web page `https://turingmachinesimulator.com/` provides a programmable Turing machine. Here you can determine the number of tapes used, the vocabulary, etc. One difference with the model we considered is that the tape is infinite in both directions.

In this problem, you are asked to create a program for this Turing machine simulator that decides the following language:

$$L = \{\ w \in \{0,1\}^* \ \mid\ w \text{ has the same number of 0s and 1s }\}.$$

You should submit a printout of your program, preferably in .txt format. You should also give a high level description explaining the logic of your program, and what each set of states is supposed to do. Finally, give a rough analysis of the running time of your machine on an input of length n using big-O notation.

3 A simple PTIME algorithm [2 points]

A **square** in an undirected graph G is an undirected 4-clique in G. Show that the following language belongs to PTIME:

$$\text{SQUARES} = \{\ \langle G \rangle \ \mid\ G \text{ contains a square }\}.$$

Note that you do not need to construct a Turing machine deciding this language, a high level description will do.