



PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory, Semester I 2017 - IIC3242

Homework 2

Deadline: Tuesday, April 10, 2018

1 Isomorphic subgraphs [15 points]

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two undirected graphs. We will say that G_1 contains a subgraph $H = (V_H, E_H)$ that is *isomorphic* to G_2 if:

- $V_H \subseteq V_1$
- $E_H \subseteq E_1$
- $|V_H| = |V_2|$
- There exists a function $f : V_2 \rightarrow V_H$ such that:
 - f is injective
 - $\{v, v'\} \in E_2$ if and only if $\{f(v), f(v')\} \in E_H$.

We define the following language:

$\text{SUBGRAPH ISOMORPHISM} = \{ \langle G_1, G_2 \rangle \mid G_1 \text{ contains a subgraph } H \text{ isomorphic to } G_2 \}$.

Show that SUBGRAPH ISOMORPHISM is NP-complete.

2 Disconnected collections [45 points]

Let $\mathcal{C} = \{S_1, \dots, S_n\}$ be a collection of sets and ℓ a natural number. We will say that \mathcal{C} is *ℓ -disconnected* if it contains ℓ different sets that are mutually disjoint. That is, we are asking if there exist numbers $i_1, \dots, i_\ell \in \{1, 2, \dots, n\}$ such that for all $j, k \in \{1, \dots, \ell\}$ it holds that $S_{i_j} \cap S_{i_k} = \emptyset$, whenever $j \neq k$.

In this problem we will consider the following language:

$\text{DISCONNECTED COLLECTION} = \{ \langle \mathcal{C} = \{S_1, \dots, S_n\}, \ell \rangle \mid \mathcal{C} \text{ contains } \ell \text{ different mutually disjoint sets} \}$.

Show that DISCONNECTED COLLECTION is NP-complete. For hardness use CLIQUE.