



PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory - IIC3242

Homework 3

Deadline: Friday, 17th of May, 2019

1 \neq -3SAT [2 points]

Let φ be a 3CNF formula. A satisfying \neq -assignment for φ is an assignment that, for each clause $(a \vee b \vee c)$ of φ makes at least one literal true, and at least one literal false. For example if $\varphi = (x \vee \neg y \vee z) \wedge (x \vee y \vee z)$, then $v_1(x) = v_1(y) = 1$ and $v_1(z) = 0$ is a satisfying \neq -assignment for φ , while $v_2(x) = v_2(y) = v_2(z) = 1$ is not a satisfying \neq -assignment for φ , although it is a satisfying assignment for φ . Let:

$$\neq\text{-3SAT} = \{ \langle \varphi \rangle \mid \varphi \text{ is a 3CNF formula that has a satisfying } \neq\text{-assignment} \}.$$

Show that \neq -3SAT is NP-complete.

2 An upper bound for wild animals [1 point]

We call a sequence of words w_1, w_2, \dots, w_k a *rainworm*, if for all $i = 1 \dots k - 1$, it holds that w_i and w_{i+1} differ in only one letter (and are of the same length). For example the sequence of words: “feed, deed, deer, dear, bear, beer” is a rainworm starting with the word “feed”, and ending with the work “beer”. Let A be the following language:

$$A = \{ \langle D, s, t \rangle \mid \text{where } D \text{ is a deterministic automaton such that } L(D) \\ \text{contains a rainworm starting in } s \text{ and ending in } t \}.$$

Show that A is in PSPACE. Note that you only have to show the upper bound.

3 Completeness in NLOGSPACE [3 points]

A directed graph G is called strongly connected if for every two nodes u, v in G , there is a (directed) path from u to v . Let:

$$\text{STRONGLY CONNECTED} = \{ \langle G \rangle \mid G \text{ is a strongly connected graph} \}.$$

Show that STRONGLY CONNECTED is an NLOGSPACE-complete problem. You need to show both the upper and the lower bound. There is no need to write down the exact Turing machine deciding this. A simple description of what such a machine does and what information it stores on its tapes will do.