PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory - IIC3242
Homework 5
Deadline: Friday, June 21, 2019

1 The first two levels of PH [6 points]

Let φ be a propositional formula *in conjunctive normal form*, and σ a valuation. We say that σ is **optimal for** φ, if the number of clauses of φ satisfied by σ is k, and there is no valuation σ' that satisfies $k+1$ or more clauses of φ. Note that if the entire formula φ is satisfiable, then any assignment satisfying it is optimal. The more interesting formulas are the ones not satisfiable.
As an example, take the formula $\varphi = (x \vee y \vee z) \wedge (\neg x \vee \neg y) \wedge (x \vee \neg y) \wedge (\neg x \vee y) \wedge (x \vee y)$. Here we have that a valuation σ with $\sigma(z) = 1$ is optimal, as it always makes 4 clauses true (irrespective of the values assigned to x and y). On the other hand, the valuation $\sigma(x) = \sigma(y) = \sigma(z) = 0$ makes only 3 clauses true.
Consider the following problem:

$$MAX - SAT = \{(\varphi, k) : \varphi \text{ is a formula, and every optimal assignment for } \varphi \text{ satisfies precisely } k \text{ clauses}\}.$$

a) Show that $MAX - SAT \in \Sigma_2^P$. [2 point]

b) Show that $MAX - SAT \in$ NP if and only if NP = co-NP. [4 points]